# What can we learn from TMDs measurements?

Alessandro Bacchetta Jefferson Lab

N. Isgur Fellowship

•3D structure of the nucleon in momentum space

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Orbital angular momentum

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Factorization

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Many interesting topics will be left out

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Many topics were already touched in the talks by M. Burkardt, M. Anselmino, F. Yuan, R. Joosten, N. Makins and several others

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A consensus on the relevance of TMDs is growing

#### Transverse Momentum Distributions (TMDs)

$$f_1^q(x, p_T^2) = \int \frac{d\xi^- d^2 \xi_T}{16\pi^3} e^{ip \cdot \xi} \langle P | \bar{\psi}^q(0) U_{[0,\xi]} \gamma^+ \psi^q(\xi) | P \rangle \bigg|_{\xi^+ = 0}$$

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$$f_1(x, p_T^2) = \frac{1}{16\pi^2} \left( |\psi_+^+(x, p_T)|^2 + |\psi_-^+(x, p_T)|^2 \right)$$

# Transverse Momentum Distributions

see e.g. A.B., Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

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		quark pol.			
		U	L	Т	
indinani	U	$f_1$		$h_1^\perp$	
	L		$g_{1L}$	$h_{1L}^{\perp}$	
	Т	$f_{1T}^{\perp}$	$g_{1T}$	$h_1,  \overline{h_{1T}^\perp}$	

Twist-2 TMDs

TMDs in black survive transverse-momentum integration TMDs in red are T-odd

nucleon pol

# **Transverse Momentum Distributions**

see e.g. A.B., Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)



Twist-2 TMDs

Twist-3 TMDs

TMDs in black survive transverse-momentum integration TMDs in red are T-odd

> For effects related to twist-3 TMDs, see talks by M. Burkardt, F. Giordano, M. Aghasyan, K. Tanaka, Y. Koike...

# Unpolarized TMDs

# Transverse momentum distributions

 $xf_1^u(x)$ 



# Transverse momentum distributions

 $xf_1^u(x)$ 







A.B., F. Conti, M. Radici, arXiv:0807.0323 see also talk by B. Pasquini

# Nucleon tomography in momentum space







Simple model calculations suggests



Simple model calculations suggests

• *x*-dependence



Simple model calculations suggests

- *x*-dependence
- flavor dependence



Simple model calculations suggests

- *x*-dependence
- flavor dependence
- deviation from a simple Gaussian



# Extractions from experiments

Drell-Yan

 $\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2 f_1^q(x, p_T^2) \otimes f_1^{\bar{q}}(\bar{x}, \bar{p}_T^2)$ 

# Extractions from experiments

$$\begin{array}{ll} \mbox{Drell-Yan} & \displaystyle \frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2 \, f_1^q(x,p_T^2) \otimes f_1^{\bar{q}}(\bar{x},\bar{p}_T^2) \\ \\ \mbox{Semi-inclusive} & \displaystyle \frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2 \, f_1^q(x,p_T^2) \otimes D_1^q(z,k_T^2) \end{array}$$

q

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electron-positron annihilation

$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2 D_1^q(z, k_T^2) \otimes D_1^{\bar{q}}(\bar{z}, \bar{k}_T^2)$$

# Analyses of Drell-Yan data



.....







# Analyses of Drell-Yan data











Resummation typically gives larger transverse momentum (requires smaller intrinsic transverse momentum) and a specific dependence on *Q* Even data at Tevatron can be described!

#### SIDIS data with hadron identification



JLab Hall C, Mkrtchyan et al., PLB665 (08)

# SIDIS data with hadron identification


Need more unpolarized measurements (SIDIS with hadron identification, electron-positron annihilation...)

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Try to abandon simple Gaussians

 Need more unpolarized measurements (SIDIS with hadron identification, electron-positron annihilation...)
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Try to abandon simple Gaussians
Use resummation

# Orbital angular momentum

# Orbital angular momentum in atoms

Hidrogen atom wavefunctions in momentum space



Vos, McCarthy, Am. J. Phys. 65 (97), 544

# Orbital angular momentum in atoms

Hidrogen atom wavefunctions in momentum space



 In atomic physics, wavefunctions with orbital angular momentum have distinct shapes

Vos, McCarthy, Am. J. Phys. 65 (97), 544

# Orbital angular momentum in atoms



- In atomic physics, wavefunctions with orbital angular momentum have distinct shapes
- The most direct visualization of these shapes is provided by scattering experiments and is in momentum space

Vos, McCarthy, Am. J. Phys. 65 (97), 544

$$f_1(x, p_T^2) = |\psi_{s-\text{wave}}|^2 + |\psi_{p-\text{wave}}|^2 + \dots$$

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Turning down of TMDs can be generated only by contributions of wavefunctions with nonzero orbital angular momentum



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Twist-2 TMDs



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# Signs of OAM in polarized TMDs



A.B., F. Conti, M. Radici, arXiv:0807.0323

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#### Longitudinal polarization measurements



Χ

see talk by P. Bosted

#### Longitudinal polarization measurements



Χ

 Non-flat behavior indicates that polarized TMDs are different from unpolarized ones

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#### Longitudinal polarization measurements



X

 Non-flat behavior indicates that polarized TMDs are different from unpolarized ones

Non-monotonic behavior is a sign of orbital angular momentum

see talk by P. Bosted

quark pol. Τ U L nucleon pol.  $h_1^\perp$ U  $f_1$  $h_{1L}^{\perp}$ L  $g_{1L}$  $h_1, h_{1T}^{\perp}$  $f_{1T}^{\perp}$ Т  $g_{1T}$ 

Twist-2 TMDs



Twist-2 TMDs

• All off-diagonal TMDs vanish in the absence of orbital angular momentum



Twist-2 TMDs

see also talks by B. Pasquini, P. Zavada

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 In general, quantitative relations between TMDs and orbital angular momentum are model-dependent



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# Sivers function



A.B., F. Conti, M. Radici, arXiv:0807.0323

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A.B., F. Conti, M. Radici, arXiv:0807.0323



QCDSF/UKQCD, PRL 98 (07)

$$f_{1T}^{\perp} = \frac{1}{16\pi^3} \operatorname{Im}\left[(\psi_+^+)^* \psi_+^- + (\psi_-^+)^* \psi_-^-\right]$$

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$$E(x,0,0) = \lim_{q_T \to 0} \left( -\frac{1}{q_x - iq_y} \frac{1}{16\pi^3} \left[ (\psi_+^+)^* \psi_+^- + (\psi_-^+)^* \psi_-^- \right] \right)$$

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$$J_q = \int_0^1 dx \, x \left( H_q(x, 0, 0) + E_q(x, 0, 0) \right) \quad \text{Ji's sum rule}$$

Model statement

$$(1-x)f_{1T}^{\perp q}(x) = -\frac{3}{2}MC_F\alpha_S E^q(x,0,0)$$
$$\int_0^1 dx(1-x)f_{1T}^{\perp q}(x) = -\frac{3}{2}MC_F\alpha_S \kappa^q$$

Burkardt, Hwang, PRD69 (04) Lu, Schmidt, PRD75 (07) A.B., F. Conti, M. Radici, arXiv:0807.0323

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$$k^u = 1.67$$
$$k^d = -2.03$$

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Anselmino et al., 0805.2677, Arnold et al. , 0805.2137

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The relation is not general

see talk by S. Meissner
### Sivers function and OAM



$$f_{1T}^{\perp q}(x) = -f(x) E^{q}(x, 0, 0)$$
  
$$f_{1T}^{\perp g}(x) = -f'(x) E^{g}(x, 0, 0)$$

### Sivers function and OAM



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• Can the Sivers measurements provide an effective way to do a flavor decomposition of the anomalous magnetic moment?

### Sivers function and OAM



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• Can the Sivers measurements provide an effective way to do a flavor decomposition of the anomalous magnetic moment?

• Can it become one of the most important sources of information also on gluon angular momentum?

Anselmino et al., 0805.2677, see talk by A. Prokudin







Worm gear





# Factorization

$$\Phi_{ij}(x, \mathbf{p_T}) = \int \frac{d\xi^- d^2 \xi_T}{8\pi^3} e^{ip \cdot \xi} \langle P | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | P \rangle \bigg|_{\xi^+ = 0}$$

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SIDIS
$$\underbrace{\xi_T}_{\xi^-} \underbrace{U_{[+]}}_{U_{[+]}} U_{[+]}$$

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 TMDs (even the unpolarized ones) are not the same in the various processes

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Maybe the universality-breaking terms are small

Collins, PLB 536 (02) Bomhof, Mulders, Pijlman, PLB 596 (04) A.B., Bomhof, Mulders, Pijlman, PRD 72 (05) Collins, Qiu, PRD 75 (07) Vogelsang, Yuan, PRD76 (07)

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In pp to hadrons, we can deal with non-universality when measuring weighted asymmetries and adding some color factors

In unweighted observables, maybe only a manageable number of distinct correlators is needed RHIC measurements are vital to check and

 Maybe the universality-breaking term understand these issues

Jomhof, Mulders, Pijlman, PLB 596 (04) A.B., Bomhof, Mulders, Pijlman, PRD 72 (05) Collins, Qiu, PRD 75 (07) Vogelsang, Yuan, PRD76 (07)



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TMDs pose some nice challenges from the theoretical point of view