Spin Content of the Nucleon

or

What We’ve Learned from Polarized Electron Scattering

Karl J. Slifer
University of New Hampshire
Spin Content of the Nucleon

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What We’ve Learned from Polarized Electron Scattering,

I’ve

(in the last few months)

Karl J. Slifer
University of New Hampshire
Burkhardt—Cottingham Sum Rule
What does the JLab data tell us?
Is it enough to make a definitive statement?

Higher Twist Measurements at Jlab

Target Mass Corrections
impact on the clean extraction of Higher Twist
When we add spin degrees of freedom to the target and beam, 2 additional SF needed.

\[
\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right] + \gamma g_1(x, Q^2) + \delta g_2(x, Q^2)
\]

all four SF needed for a complete description of nucleon structure
Parton Model

Interpretation of the Structure Functions

Impulse Approximation in DIS
no time for interaction between partons

distributions of quark momentum and spin in the nucleon.

\[ F_1(x) = \frac{1}{2} \sum e_i^2 \left[ q_i(x) + \bar{q}_i(x) \right] \]

runs over all quark flavors

\[ F_2(x) = 2xF_1(x) \]

\[ g_1(x) = \frac{1}{2} \sum e_i^2 \Delta q_i(x) \]

\[ g_2(x) = ??? \]
Burkhardt-Cottingham Sum Rule

\[ \int_{0}^{1} g_{2}(x, Q^{2}) \, dx = 0 \]

H. Burkhardt and W. N. Cottingham

Relies on the virtual Compton scattering amplitude \( S_{2} \) falling to zero faster than \( 1/\nu \) as \( \nu \to \infty \)

Discussion of possible causes of violations


“If it holds for one \( Q^{2} \) it holds for all”
BC Sum Rule

Existing World Data on $\Gamma_2$

for Proton Neutron and $^3$He

BLACK : E94010. (Hall A, $^3$He)

BROWN : E155. (SLAC NH3, $^6$LiD)

Note:
SLAC “Measured” = 0.02 < x < 0.8
JLAB “Measured” ≈ Resonance Region
BC Sum Rule

BRAND NEW DATA!

Very Preliminary

RED: RSS. (Hall C, NH$_3$, ND$_3$)
K. Slifer, O. Rondon et al. in preparation

courtesy of P. Solvignon

BLUE: E01-012. (Hall A, $^3$He)
courtesy of V. Sulkosky

GREEN: E97-110. (Hall A, $^3$He)

Thanks also to the spokesmen of these experiments!

RSS: Mark Jones, Oscar Rondon
E01-012: N. Liyanage, J.P.Chen, Seonho Choi
SaGDH: J.P. Chen, A. Deur, F. Garabaldi
Good agreement with MAID model of resonance region for JLab data

very prelim
BC Sum Rule

Neutron results around $Q^2=1.3$ GeV$^2$ from 2 very different experiments:

- **RSS** in Hall C: Neutron from ND$_3$ & NH$_3$
- **E01-012** in Hall A: Neutron from $^3$He

Excellent agreement!
BC Sum Rule

Good overlap at low $Q^2$ of the old and new neutron data

E94010 : Hall A $^3$He old

E97-110 Hall A $^3$He new
BC Sum Rule

\[ P \]

\[ N \]

\[ 3 \text{He} \]

Measured $x$

Difference between $N$ and $^3\text{He}$ Sum rules

\[ ^3\text{He} \approx \text{P}_n\text{Neutron} + 2\text{P}_p\text{Proton} \]

\[ \pi \text{ threshold} \]

Note: $^3\text{He}$ requires use of nuclear elastic
BC Sum Rule

\[ \int_{0}^{1} g_2(x, Q^2) dx = 0 \]

**BC** = **RES**+**DIS**+**ELASTIC**

**“RES”:** Here refers to measured x-range

**“DIS”:** refers to unmeasured low x part of the integral. Not strictly Deep Inelastic Scattering due to low \( Q^2 \)

Assume Leading Twist Behaviour

**Elastic:** From well know FFs (<5%)
BC Sum Rule

$$BC = RES + DIS + ELASTIC$$

"RES": Here refers to measured x-range

"DIS": refers to unmeasured low x part of the integral. Not strictly Deep Inelastic Scattering due to low $Q^2$

Assume Leading Twist Behaviour

Elastic: From well know FFs (<5%)
BC Sum Rule

- BC satisfied w/in errors for JLab Proton
  - 2.8σ violation seen in SLAC data

- BC satisfied w/in errors for Neutron
  - (But just barely in vicinity of $Q^2 = 1$!)

- BC satisfied w/in errors for $^3\text{He}$
Proton $g_{2p}$ still relatively unknown for such a fundamental quantity.

Need more high quality data like RSS

**Upcoming Experiments**

**Sane**: setting up now!

$2.3 < Q^2 < 6 \text{ GeV}^2$

"g2p" in Hall A, 2011

$0.015 < Q^2 < 0.4 \text{ GeV}^2$
What can BC tell us about Low-X?

Alternatively, if we assume BC holds we can learn something about the unmeasured part of Integral

\[ \int_0^1 g_2(x, Q^2) dx = 0 \]

BC = 0 ⇒ Res + Elas + “DIS”

Unmeasured Low-x part

“DIS” = -(RES+ELAS)
What can BC tell us about Low-X?

\[ \text{DIS} = -(\text{RES} + \text{ELAS}) \]
Just on the edge of being interesting

Statistical precision allows unambiguous test, but limited by large systematics.

Highly desirable to revisit E94010 systematics. RC and DIS could perhaps be improved with newer data.

Measured $0 < x < 1$

**Neutron BC Sum**

### Standard Deviations from Zero

<table>
<thead>
<tr>
<th>$Q^2$</th>
<th>Tot $\sigma$</th>
<th>Stat $\sigma$</th>
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<tr>
<td>0.74</td>
<td>1.7</td>
<td>6.6</td>
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<tr>
<td>0.9</td>
<td>2.0</td>
<td>7.9</td>
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<td>1.2</td>
<td>2.4</td>
<td>2.9</td>
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<tr>
<td>1.8</td>
<td>1.9</td>
<td>2.2</td>
</tr>
<tr>
<td>2.4</td>
<td>1.8</td>
<td>2.2</td>
</tr>
</tbody>
</table>

**BC Sum Rule**
Higher Moment

Cornwall Norton Moment

\[
I(Q^2) = 2 \int_0^{1-\varepsilon} x^2 (2g_1 + 3g_2) \, dx
\]

\(I(Q^2) \neq \) the twist-3 matrix element but very interesting all the same.

More on this later...
What's happening at large $Q^2$?

**Existing World Data on $I(Q^2)$**

**BLACK**: E94010

**BROWN**: E155

**RED**: RSS.

**Magenta**: E99-117
I(Q^2)

MAID Model

Very Preliminary

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K. Slifer, O. Rondon et al. in preparation

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$I(Q^2)$

$I(Q^2) \rightarrow 0$ for $Q^2 = 0$ and $Q^2 = \infty$

largest around $Q^2 = 1$

Not too useful to the OPE in this region, but excellent gauge of “QCD complexity”

(i.e. where’s the most difficult place to make any meaningful QCD calculation?)
I(Q^2)

Shaded Region: Estimate of I(Q^2)

Large uncertainty due to lack of knowledge of g2p
I(Q²)

Upcoming 6 GeV Experiments

Sane: Fall 2008
2.3 < Q² < 6 GeV²

“g2p” in Hall A, 2011
0.015 < Q² < 0.4 GeV²

“d2n” in Hall A, 2009
Q² = 3 GeV²
Operator Product Expansion

Expansion of SF moments in powers of $1/Q^2$ ("twist")

Example:

$$\Gamma_1(Q^2) = \int_0^1 g_1(x,Q^2) dx = \sum_{\tau=2,4,...} \frac{\mu_2(Q^2)}{Q^{\tau-2}}$$

$$\mu_4 = \frac{1}{9} M^2 (\tilde{a}_2 + 4 \tilde{d}_2 + 4 \tilde{f}_2)$$

Lowest order (twist-2) maps to the successful parts of the parton model.

Higher twists arise from non-perturbative multiparton interactions.
Cornwall-Norton Moments

\[
I(Q^2) = 2 \int_0^{1-\varepsilon} x^2 (2g_1 + 3g_2) dx
\]

\[
= \tilde{d}_2(Q^2) + \theta \left( \frac{M^2}{Q^2} \right)
\]

Typical method of extracting twist-3 matrix element

But completely ignores TMC!

Very significant below \( Q^2 \approx 5 \)

Y.B. Dong PRC 77(2008) 015201

Y.B.Dong PLB 653,(2007)18
Nachtmann Moments

Nachtmann Moments:

\[
M_2^3(Q^2) = \int_0^1 dx \left( \frac{\xi^4}{x^2} \right) \left\{ \frac{x}{\xi} g_1 + \left[ \frac{3}{2} \left( \frac{x}{\xi} \right)^2 - \frac{3}{4} \frac{M^2}{Q^2} x^2 \right] g_2 \right\}
\]

\[
= \tilde{d}_2 \frac{d_2}{2}
\]


Generalization of CN moments to protect from the TMC

\[
\frac{M^2}{Q^2} \rightarrow 0 \quad M_2^3 \rightarrow \int x^2 (2g_1 + 3g_2) dx
\]

Reduces to familiar form

Not a new idea, but difficult to implement unless \( g_2 \) measured simultaneously with \( g_1 \)
Quantifying Size of TMC

\[ R(Q^2) = \frac{2M_2^3(Q^2)}{I(Q^2)} \]

- \( R \to 1 \) in case of vanishing nucleon mass

- \( R \) always less than 1 \( \Rightarrow I(Q^2) \) overestimates twist-3

Target Mass Corrections must be applied in order to obtain clean dynamical Twist-3
Generalization of $\Gamma_1$

\[
M_1^1(Q^2) = \int_0^1 dx \left( \frac{x}{\xi} \right)^2 \left\{ \left[ \frac{x}{\xi} - \frac{1}{9} \left( \frac{M}{Q} \right)^2 \frac{x}{\xi} \right] g_1 - \left( \frac{M}{Q} \right)^2 x^2 \frac{4}{3} g_2 \right\}
\]

\[
\frac{M^2}{Q^2} \to 0 \quad M_1^1 \to \Gamma_1
\]


Osipenko et al. PRD 71, 054007 (2005)

Global analysis of $g_1p$ data. Allows to cleanly extract leading twist term.

TMC not as large as for $I(Q^2)$
Burkhardt–Cottingham Sum Rule
Good coverage for Neutron. Proton $g_2p$ is still relatively unknown.

Data seems to validate BC, but at the $2.5\sigma$ level around $Q^2=1$
Important to update the systematics of the old experiments

Assuming BC holds, we can use JLab data to say something about low-$x$.

Target Mass Effects
TMC are significant at JLab kinematics

Nachtmann moments protect the SSF from TMC

Must use Nachtmann Moments in order to cleanly extract Higher twists

JLab 6 GeV Program
Still lots of Good Physics to be completed before the upgrade.
Backups
**REFERENCES**

**BLACK**: E94010. (Hall A, $^3$He)


**RED**: RSS. (Hall C, NH$_3$,ND$_3$)

Wesselman, Slifer, Tajima *et al.*  
Slifer, Rondon *et al.* in preparation

**BROWN**: E155. (SLAC NH$_3$,LiD)


**Magenta** E99-117(Hall A, $^3$He)


**BLUE**: E01-012. (Hall A, $^3$He)

P. Solvignon *et al.* arXiv:0803.3845  (PRL accepted)  
P. Solvignon *et al.* in preparation

**SHADED**: Theory

Osipenko *et al.* PRD. 71 (2005) 054007
**BC Sum Rule**

**Unmeasured Contributions**

**Low-X Estimate**

Assume $g_2 = g_2^{ww}$ at low $x$.

Supported by RSS data

15% variation seen depending on choice of $g_1$ used.

**ELASTIC: $X=1$**

\[
g_1^{el}(x, Q^2) = \delta(x - 1)G_M(Q^2)\frac{G_E(Q^2) + \tau G_M(Q^2)}{2(1 + \tau)}
\]

\[
g_2^{el}(x, Q^2) = \delta(x - 1)\tau G_M(Q^2)\frac{G_E(Q^2) - G_M(Q^2)}{2(1 + \tau)}
\]

(Form Factor uncertainties less than 5%)
Summary

- Hydrogen Hyperfine Structure
- $d^p_2(Q^2)$: Measure of QCD complexity
- Systematic uncertainty in Measurements of $g^p_1$
- Extended GDH SUM
- Resonance Structure $\Delta(1232)$
- Spin Polarizability $\delta_{LT}(Q^2)$
- Ideal place to test $\chi$PT calcs
JLab Kinematic Coverage

Overview of available kinematic range at JLab

Uniquely positioned to provide data in transition region of QCD
Target Mass Corrections

Purely kinematic effects from finite value of $4M^2x^2/Q^2$

\[ g_1(x, Q^2) = g_1(x, Q^2, M = 0) \]

\[ + \frac{M}{Q^2} g_1^{(1)TM C}(x, Q^2) \]

\[ + \frac{h(x, Q^2)}{Q^2} + \theta(1/Q^4) \]

From PQCD

Purely kinematical

Higher twist

\[ \int_0^1 x^2 g_1(x, Q^2)dx = \frac{1}{2} \tilde{a}_2 + \theta \left( \frac{M^2}{Q^2} \right) \]

\[ \int_0^1 x^2 g_2(x, Q^2)dx = \frac{1}{3} (\tilde{a}_2 - \tilde{a}_2) + \theta \left( \frac{M^2}{Q^2} \right) \]
Q^2 evolution predicted well by PQCD and Chiral Soliton models.

WGR: PRD55 (1997) 6910
Wakamatsu: PLB 487(2000)118
QCDSF: PRD63, 074506(2001)
$g_2$ Structure Function

Wandzura-Wilczek relation

$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy$

Leading twist determined entirely by $g_1$

$g_2 = g_2^{WW} + \bar{g}_2$

$g_2$ doesn’t exist in Parton Model. Good quantity to study higher twist
Inclusive Cross Section

\[ \frac{d^2 \sigma}{d\Omega dE'} = \sigma_{Mott} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right] \]

deviation from point-like behavior characterized by the Structure Functions